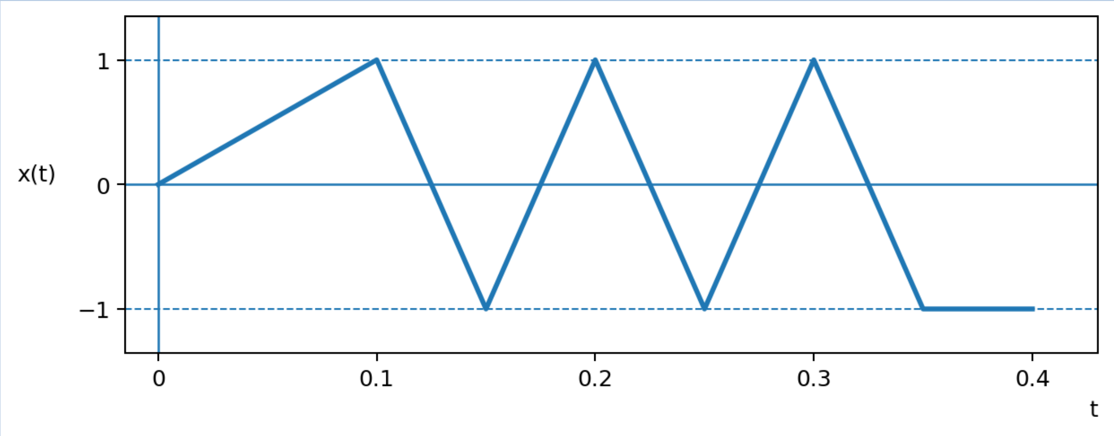
Periodic and Non-Periodic Signals

Questions, step-by-step solutions, examples and homework

Q1. Fundamental Period of a Triangular Wave

Question: What is the fundamental period of the waveform shown below?



Solution:

The given signal is a triangular wave which starts at t = 0 and again starts at t = 0.2.

$$\therefore \text{ Period, } T = 0.2 - 0 = 0.2 \text{ sec}$$

Fundamental frequency:

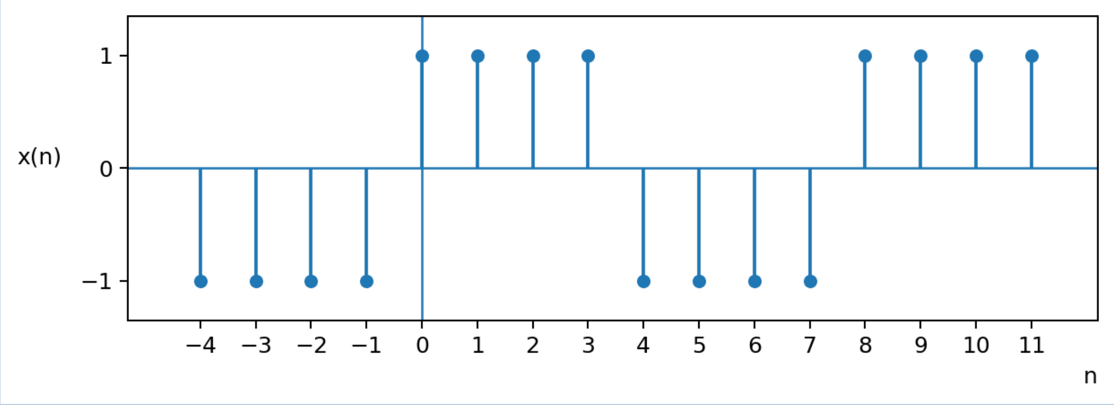
$$f = 1/T = 1/0.2 = 5 \text{ Hz}$$

If we want to express fundamental frequency in rad/sec:

$$\omega = 2\pi f = 2\pi \times 5 = 10\pi = 31.4 \text{ rad/sec}$$

Q2. Fundamental Frequency of a Discrete-Time Square Wave

Question: What is the fundamental frequency of the discrete-time square wave shown below?



Solution:

The above DT signal starts at $n = 0$ and again starts at $N = 8$.

$$\therefore \textbf{ Fundamental period, } N = 8 - 0 = 8 \textbf{ samples}$$

$$\textbf{Fundamental frequency, } f = 1/N = 1/8 \text{ Hz}$$

$$\textbf{Fundamental frequency in radians: } \omega = 2\pi f = 2\pi/N = 2\pi/8 = \pi/4 \text{ radians}$$

Q3. Check Periodicity and Find the Fundamental Period

Question: Determine whether the continuous-time signal $x(t) = [\cos(2\pi t)]^2$ is periodic or not. If periodic, find the fundamental period T.

Use formula:

$\cos^2 x = (1 + \cos 2x)/2$

Solution:

$$x(t) = [\cos(2\pi t)]^2$$

$$= \cos^2(2\pi t)$$

$$= [1 + \cos(4\pi t)]/2$$

$$\mathbf{= 1/2 + 1/2 \cos(4\pi t)}$$

The first term is constant, so its frequency is zero. The second term determines the periodic repetition.

Compare the second term with $A \cos(\omega t) = A \cos(2\pi f t)$.

$$2\pi f t = 4\pi t$$

$$\therefore f = 2 \text{ Hz}$$

$$\textbf{Therefore, } T = 1/f = 1/2 = 0.5 \textbf{ sec}$$

Checking the periodicity condition: If a signal is periodic, $x(t) = x(t + T)$ must be satisfied.

Now, $x(t + 0.5) = [\cos 2\pi(t + 0.5)]^2$

$$= \cos^2\{2\pi(t + 0.5)\}$$

$$= [1 + \cos\{4\pi(t + 0.5)\}]/2$$

$$= 1/2 + 1/2 \cos(4\pi t + 2\pi)$$

Useful identities:

$\sin(n\pi) = 0$

$\cos(2\pi) = 1$

$\sin(2\pi) = 0$

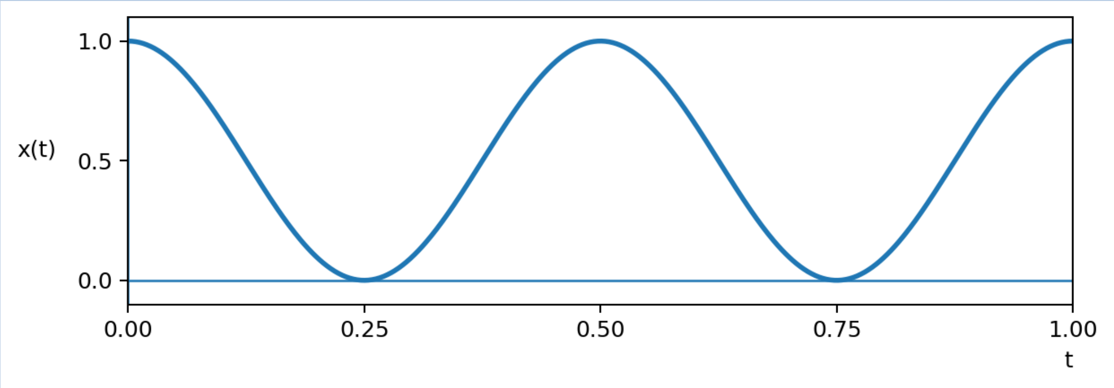
$\cos(A + B) = \cos A \cos B - \sin A \sin B$

$$= 1/2 + 1/2[\cos(4\pi t)\cos(2\pi) - \sin(4\pi t)\sin(2\pi)]$$

$$= 1/2 + 1/2 \cos(4\pi t)$$

$$= \cos^2(2\pi t) = [\cos(2\pi t)]^2 = x(t)$$

So, we proved that $x(t + 0.5) = x(t)$. Hence the given signal is periodic with fundamental period $T = 0.5$ seconds.



Q4. Periodicity of Continuous-Time and Discrete-Time Signals

Determine whether the following signals are periodic or not. If periodic, determine the fundamental period.

(i) $\cos(0.01\pi n)$	(ii) $\cos(3\pi n)$	(iii) $\sin(\pi n + 0.2n)$
Compare with $\cos(2\pi fn)$. $2\pi fn = 0.01\pi n$ $\therefore f = 0.01/2 = 1/200 = K/N$. Since $K = 1$ and $N = 200$ are integers, the signal is periodic. Fundamental period: $N = 200$.	Compare with $\cos(2\pi fn)$. $2\pi fn = 3\pi n$ $\therefore f = 3/2 = K/N$. This is a rational number, so the signal is periodic. Fundamental period: $N = 2$.	Compare with $\sin(2\pi fn + \phi)$. $2\pi fn + \phi = 0.2n + \pi$. The phase is π, so it does not affect periodicity. $2\pi fn = 0.2n$ $\therefore f = 0.2/(2\pi) = 1/(10\pi)$, which is irrational. Therefore, the signal is non-periodic.

Q5. Check Periodicity of DT Signals

(i) $\cos(2\pi n/5) + \cos(2\pi n/7)$

The given signal is the addition of two DT signals. Find the period of each signal separately.

$$\text{Let } x_1(n) = \cos(2\pi n/5), \quad x_2(n) = \cos(2\pi n/7)$$

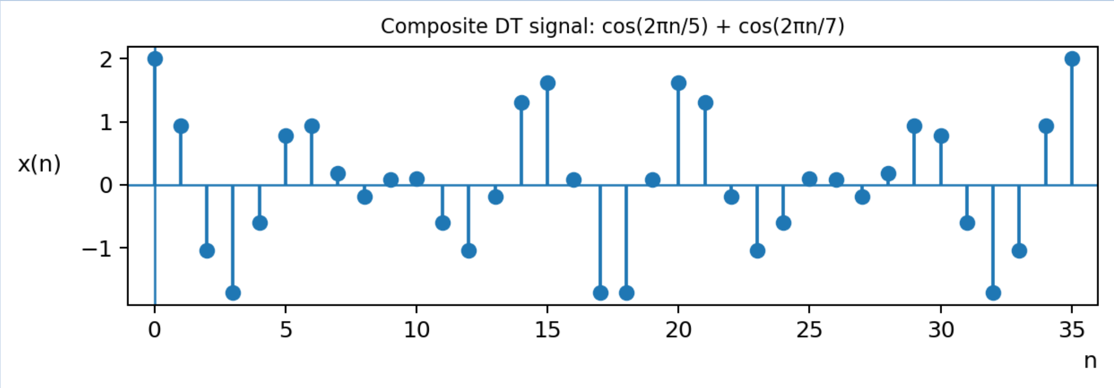
$$\text{For } x_1(n):\ 2\pi f_1 n = 2\pi n/5 \Rightarrow f_1 = 1/5 = K_1/N_1 \Rightarrow N_1 = 5$$

$$\text{For } x_2(n):\ 2\pi f_2 n = 2\pi n/7 \Rightarrow f_2 = 1/7 = K_2/N_2 \Rightarrow N_2 = 7$$

$N_1/N_2 = 5/7$, a ratio of two integers. Hence the composite signal is periodic.

Period of composite signal = $\mathrm{LCM}(N_1, N_2) = \mathrm{LCM}(5, 7) = 35$

$$\therefore\ N = 35$$



Q5 (ii). Product of Two DT Signals

Given signal: $\cos(n/8) \cdot \cos(n\pi/8)$ is the product of two signals.

$$\text{Let } x_1(n) = \cos(n/8), \quad x_2(n) = \cos(n\pi/8)$$

$$\text{For } x_1(n): \; 2\pi f_1 n = n/8 \;\Rightarrow\; f_1 = 1/(16\pi)$$

Since $1/(16\pi)$ is not rational, $x_1(n)$ is non-periodic.

$$\text{For } x_2(n): \; 2\pi f_2 n = n\pi/8 \;\Rightarrow\; f_2 = 1/16 = K_2/N_2$$

Since 1/16 is rational, $x_2(n)$ is periodic.

Important observation:

In this example, the product contains a non-periodic factor, so the given product is non-periodic.

Hence, $\cos(n/8) \cdot \cos(n\pi/8)$ is non-periodic.

Important Note on Addition of Periodic Signals

NOTE:

Addition of more than two periodic signals can be analysed by finding the ratio of the period of any one signal with the periods of the others.

If any required ratio is not a rational number, the composite signal cannot be periodic.

If the composite signal is periodic, its period can be obtained from the LCM of the corresponding integer periods (or, equivalently, from the common repetition interval).

Homework on Periodic & Non-Periodic Signals

Check periodicity for the following signals. Also find periodicity/period, wherever applicable.

Continuous-Time Signals	Discrete-Time Signals
(i) $x(t) = 2\cos(3t + \pi/4)$	(i) $x(n) = \cos(3\pi n)$
(ii) $x(t) = \cos(t + \pi/4)$	(ii) $x(n) = \cos(8\pi n/7 + 2)$
(iii) $x(t) = \cos(\pi t/3) + \sin(\pi t/4)$	(iii) $x(n) = \cos(\pi n/3) \cdot \sin(2n)$
(iv) $x(t) = \cos t + \sin(\sqrt{2}\, t)$	(iv) $x(n) = \cos(2\pi n)$
(v) $x(t) = [\sin(t - \pi/4)]^2$	(v) $x(n) = \sin(\pi + 0.2n)$
	(vi) $x(n) = e^{(j\pi n/4)}$

Quick Revision

- A continuous-time sinusoid $\cos(2\pi ft + \phi)$ is periodic when $f \neq 0$; its period is $T = 1/|f|$.
- A discrete-time sinusoid $\cos(2\pi fn + \phi)$ is periodic when f is rational. If $f = K/N$ in lowest terms, the fundamental period is N samples.
- A constant has zero frequency and is periodic with any shift; when combined with a periodic sinusoid, it does not destroy periodicity.
- For sums of periodic signals, a common period must exist. For integer sample periods, the composite period is the LCM of the individual fundamental periods.