Solved assignment problems in communication | online Request

Q1. Consider that a 100-kbps data stream is to be transmitted on a voice-grade telephone circuit (with a bandwidth of 3kHz). Is it possible to approach error-free transmission with a SNR of 10 dB? Justify your answer. If it is not possible, suggest system modifications that might be made.

Sol:

Shannon Channel Capacity Theorem

The Shannon channel capacity theorem defines the theoretical maximum bit rate (number of bits per second) for a noisy channel

Capacity $C = B \; x \; log_2(1 + \frac{S}{N})$

Where, C = Channel capacity (bps)
B = Bandwidth of signal
S = Signal power (Watt)
N = Noise power (Watt)

$$SNR = \frac{S}{N} = \frac{Signal\ Power}{Noise\ Power}\ (no\ unit)$$

Generally, SNR is expressed in decibel (dB) units. [dB = 10 $\log_{10}$ (S/N)]

Every channel has maximum data rate R. Remember that $\boldsymbol{R \leq C}$ for error-free transmission.

Given data:

R = 100 kbps
B = 3 kHz
$\frac{S}{N}$ = SNR = Signal power to Noise power ratio = 10 dB

$$\therefore\ channel\ capacity, C = B\ x\ log_2(1 + \frac{S}{N}) = 3\ x\ 10^3\ x\ \log_2\left(1 + \frac{S}{N}\right)$$

$\frac{S}{N}$ *is given in dB, we should convert into numerical value*

$$10\ dB = 10\ \log_{10} \frac{S}{N}$$

$$1 = \log_{10} \frac{S}{N}$$

$$\therefore\ \frac{S}{N} = antilog\ 1 = 10^1 = 10\ numerical\ value$$

Substitute this numerical value in C.

$$\therefore C = 3\ x\ 10^3\ x\ \log_2\left(1 + \frac{S}{N}\right)$$
$$= 3\ x\ 10^3\ x\ \log_2 11$$
$$= 3\ x\ 10^3\ x\ 3.4594$$
$$= 10378.26\ bps$$
$$= 10.378\ kbps$$

Compare system data rate with channel capacity.

- R = 100 kbps
- C = 10.378 kbps

So, here $R > C$, which is fundamentally wrong. In order to achieve error-free transmission, $\boldsymbol{R \leq C}$. Hence, it is not possible to achieve error-free transmission with given SNR = 10 dB and BW = 3kHz.

Changes need to be done in the design of communication system as shown below in order to achieve error-free transmission.

Remember TRADE-OFF between SNR and BW.

We know, $C = B\ x\ log_2(1 + \frac{S}{N})$

C is constant for a given channel. The only changing parameters are: Bandwidth (B) and SNR.

We can trade between B and SNR to achieve constant value of C.

To achieve 100 kbps without any errors, two cases can be possible:

i. We should change BW, keeping SNR constant
ii. We should change SNR, keeping BW constant

Case 1: Keep SNR constant i.e. SNR = 10dB = 10 numerical value and we know R = 100 kbps

$$\therefore 100\ kbps = B\ x\ \log_2\left(1 + \frac{S}{N}\right)$$

$$100\ kbps = B\ x\ \log_2(1 + 10)$$
$$100\ kbps = B\ x\ \log_2 11$$
$$100\ kbps = B\ x\ 3.4594$$

$$\log_2 11 = \frac{\log_{10} 11}{\log_{10} 2}$$
$$= \frac{1.04139}{0.30102999}$$
$$= 3.4594$$

$$\therefore B = \frac{100\ x\ 10^3}{3.4594} = 28906\ Hz = 28.906\ kHz$$

Case 2: Keep BW constant i.e. B = 3 kHz and we know R = 100 kbps

$$\therefore 100\ kbps = 3\ x\ 10^3\ \log_2\left(1 + \frac{S}{N}\right)$$

$$100\ kbps = 3\ x\ 10^3\ \log_2\left(1 + \frac{S}{N}\right)$$

$$Let\ \left(1 + \frac{S}{N}\right) = t$$

$$100\ kbps = 3\ x\ 10^3\ \log_2 t$$

$$\log_2 t = \frac{100\ kbps}{3\ x\ 10^3} = \frac{100\ x\ 10^3}{3\ x\ 10^3} = \frac{100}{3} = 33.33$$

$$\frac{\log_{10} t}{\log_{10} 2} = 33.33$$

$$\log_{10} t = \log_{10} 2\ x\ 33.33$$

$$\log_{10} t = 0.30102999\ x\ 33.33$$

$$\log_{10} t = 10.034$$

$$\therefore t = antilog\ (10.034) = 10^{10.034} = 10822634704$$

$$since, \left(1 + \frac{S}{N}\right) = t = 10822634704$$

$$\frac{S}{N} = 10822634703\ numerical\ value$$

$$\therefore \frac{S}{N}\ in\ dB = 10\ \log_{10}\frac{S}{N} = 10\ \log_{10} 10822634703 = 100.34\ dB$$

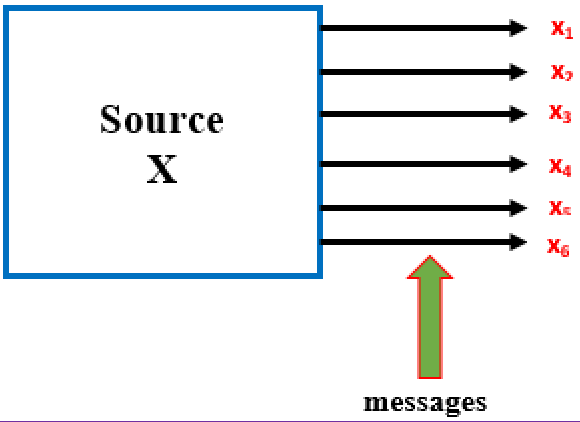
Modifications suggested are as follows:

Modification 1: SNR = 10 dB, BW = 28.906 kHz, we can achieve data rate R = 100 kbps
Modification 2: BW = 3 kHz, SNR = 100.34 dB, we can achieve data rate R = 100 kbps

Under any of above two modifications, error-free transmission is possible.

Q2. Consider a source that produces six messages with probabilities $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}$ *and* $\frac{1}{32}$. Determine the average information content in bits of a message.

Sol:



$p_1 = p(x_1) = \frac{1}{2} = 0.5$

$p_2 = p(x_2) = \frac{1}{4} = 0.25$

$p_3 = p(x_3) = \frac{1}{8} = 0.125$

$p_4 = p(x_4) = \frac{1}{16} = 0.0625$

$p_5 = p(x_5) = \frac{1}{32} = 0.03125$

$p_6 = p(x_6) = \frac{1}{32} = 0.03125$

Entropy H(X) = Average information of a source X

Entropy is defined as H(X) and is given by:

$$H(X) = \sum_{1}^{l} p(x_i)\, I(x_i) = \sum_{1}^{l} p(x_i) \log_2 \frac{1}{p(x_i)}$$

Where X = source and *l* = size of alphabet

$$H(X) = \sum_{1}^{6} P(x_i) \log_2 \frac{1}{P(x_i)}$$

$$= P(x_1)\log_2\frac{1}{P(x_1)} + P(x_2)\log_2\frac{1}{P(x_2)} + P(x_3)\log_2\frac{1}{P(x_3)} + P(x_4)\log_2\frac{1}{P(x_4)} + P(x_5)\log_2\frac{1}{P(x_5)} + P(x_6)\log_2\frac{1}{P(x_6)}$$

$$= 0.5\ \log_2 2 + 0.25\ \log_2 4 + 0.125\ \log_2 8 + 0.0625\ \log_2 16 + 0.03125\ \log_2 32 + 0.03125\ \log_2 32$$

$$= 0.5\ x\ 1 + 0.25\ x\ 2 + 0.125\ x\ 3 + 0.0625\ x\ 4 + 0.03125\ x\ 5 + 0.03125\ x\ 5$$

$$= 1.7812 \text{ bits/message}$$

- Entropy defines ultimate **data compression** of a source. Data compression provides a practical means for the efficient storage and transmission of data (text, audio, video etc.)
- Larger entropy represents larger average information.

Q3. A spacecraft at a distance of 381000 Km from the earth's surface is equipped with a transmitter with a power of 6 watt. The attenuation of the signal between the transmitter and receiver on the earth is given by $L = 150 + \log_{10}(X) dB$, where X is the distance measured in kilometres. The noise temperature power at the receiver 58 K. The gain of transmit and receive antennas are 26 dB and 50dB respectively. Calculate:

a) Signal power at the receiver input
b) Noise density (N_0) at the receiver.

Sol:

Friis transmission equation used to calculate the signal power arriving at the receiver, which is given as:

$$P_r = P_t \, G_t \, G_r \left[\frac{\lambda}{4\pi R}\right]^2$$

Where

P_r = Received power level =???
P_t = Transmit power level = 6 W
λ = Transmit wave length
G_t = Gain of the transmit antenna = 26 dB
G_r = Gain of the transmit antenna = 50 dB
R = Separation distance between antennas = 381000 Km

Also, equation for free-space loss or attenuation is given by formula:

$$L = \left[\frac{4\pi R}{\lambda}\right]^2$$

a. Signal power at the receiver input

$$\therefore Received\ power\ P_r = \frac{P_t \, G_t \, G_r}{L}$$

Given that Attenuation or Loss

$L = 150 + \log_{10}(X) dB$
$= 150 + \log_{10} 381000$
$= 150 + 5.5801$
$= 155.58$ dB

$10 \log_{10} x = 155.58$
$\log_{10} x = 15.558$
$\therefore x = antilog\ (15.558) = 10^{15.558} = 3630780547701013$

$= 3630780547701013$ numerical value

G_t = Gain of the transmit antenna = 26 dB = 398.107 numeric

$10 \log_{10} x = 26\ dB$
$\log_{10} x = 2.6$
$\therefore x = antilog\ (2.6) = 10^{2.6} = 398.107$ numeric

G_r = Gain of the receiving antenna = 50 dB = 100000 numeric

$10 \log_{10} x = 50\ dB$
$\log_{10} x = 5$
$\therefore x = antilog\ (5) = 10^5 = 100000$ numeric

$$\therefore \text{Received power } P_r = \frac{P_t\, G_t\, G_r}{L}$$

$$= \frac{6\ x\ 398.107\ x\ 100000}{3630780547701013}$$

$$= 65.7\ nWatt$$

b. Noise density (N_0) at the receiver.

We know that Signal – to – Noise density is given by equation:

$$\frac{P_r}{N_0} = \frac{P_t G_t}{\left[\frac{4\pi R}{\lambda}\right]^2}\left(\frac{G_r}{T}\right)\frac{1}{K}$$

Where T = Noise temperature in Kelvin = 58 (given)
K = Boltzmann constant = 1.38 x 10^{-23} J/degree kelvin

$$\therefore \frac{P_r}{N_0} = \frac{P_t G_t}{L}\left(\frac{G_r}{T}\right)\frac{1}{K} \quad \text{and} \quad N_0 = \frac{P_r\, L\, T\, K}{P_t G_t G_r}$$

$$\therefore \text{Noise density } N_o = \frac{65.7\ x\ 10^{-9}\ x\ 3630780547701013 \text{ x } 58 \text{ x } 1.38 \text{ x } 10^{-23}}{6\ x\ 398.107\ x\ 100000}$$

$$= 1378140169.76\ x\ 10^{-32}\ Watt/Hz$$