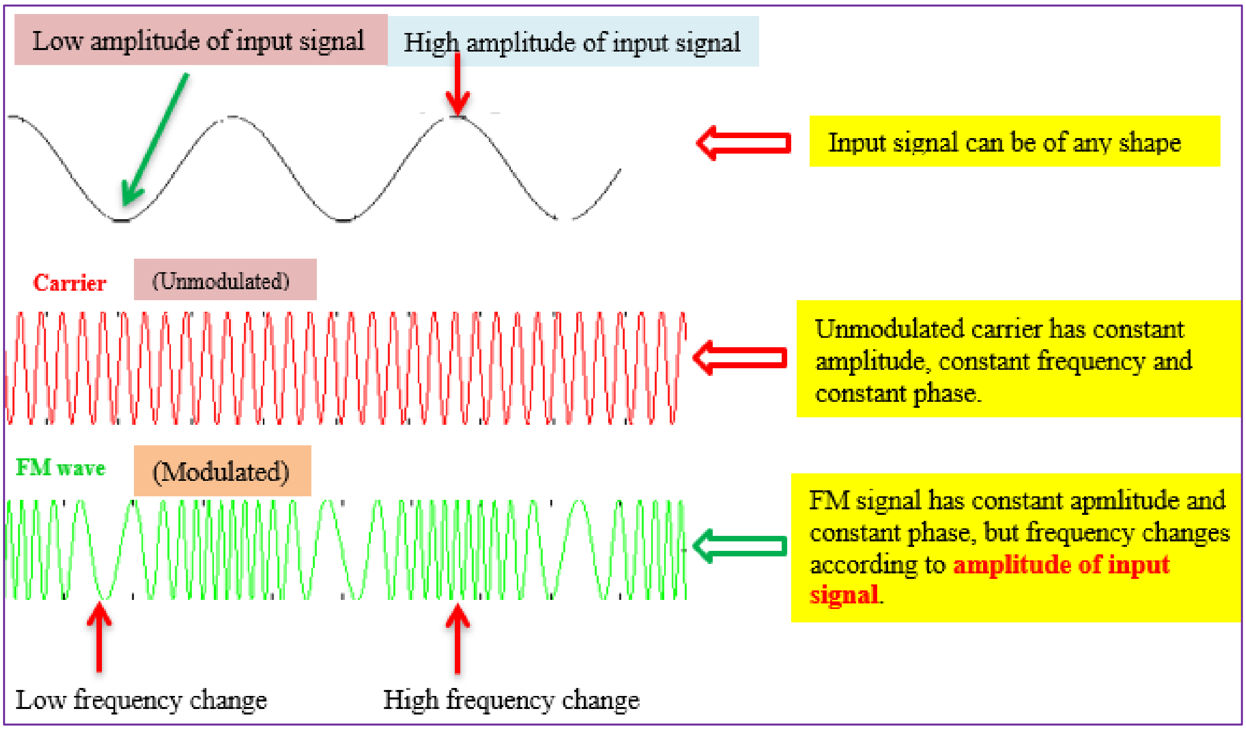
FREQUENCY MODULATION (FM)

Modulation is the process by which some characteristic of the carrier is varied in accordance with modulating (input) signal. FM is the modulation in which the instantaneous frequency of the carrier c(t) is varied linearly with message signal m(t). This means frequency of the carrier is changed according to amplitude of modulating signal. So, in FM message signal (baseband signal) changes the frequency of the carrier wave without changing its amplitude or phase angle.



Mathematical representaion of FM

Mathematical expression for a sinusoidal carrier is:

$$c(t) = A_c \cos(w_c t + \emptyset) \text{ or } A_c \sin(w_c t + \emptyset)$$

Where

$A_c = carrier amplitude$

$w_c = Angular\ frequency\ of\ carrier\ (radians/sec) = 2\pi f_c$

$f_c = carrier\ frequency\ in\ Hz$

$\emptyset = carrier\ phase$

Note: when doing AM mathematical analysis, we put $\emptyset = 0$ for simplifications

Remember that in FM, message signal m(t) changes the frequency of the carrier wave without changing its amplitude and phase.

$$Let\ carrier\ c(t) = A_c \cos(w_c t) = A_c \cos 2\pi f_c t = A_c \cos \theta_c$$

where $\theta_c = total\ phase\ angle\ of\ unmodulated\ carrier$

$$\theta_c = w_c t = 2\pi f_c\, t \quad \ldots\ldots\ldots\ldots (1)$$

Differentiating equation **1** on both sides with respect to time t

$$\frac{d}{dt}\theta_c = \frac{d}{dt}\{2\pi f_c t\} = 2\pi f_c$$

$$\therefore\ d\theta_c = 2\pi f_c dt$$

Similarly, after modulation, the modulated carrier has some angle. Let us represent this angle with θ_i. Here, i represents instantatneous value of the carrier angle.

After modulation carrier becomes

$FM(t) = A_c \cos(w_i t) = A_c \cos 2\pi f_i t = A_c \cos \theta_i$ .... (2)

f_i is the instataneous frequency of the carrier after modulation

From equation (2), we can write $\theta_i = 2\pi f_i t$ .... (3)

Differentiating equation **3** on both sides with respect to time t

$$\frac{d}{dt}\theta_i = \frac{d}{dt}\{2\pi f_i t\} = 2\pi f_i$$

$\therefore\ d\theta_i = 2\pi f_i dt$ .... (4)

Recall AM analysis: $[A_c + m(t)] \cos w_c t$

Instantaneous amplitude of the carrier

Similarly, for FM $w_i = [w_c + k_f\, m(t)]$........... (5) **THIS IS THE DEFINITION OF FM**

Instantaneous frequency of the carrier

Where $k_f = proportionality\ constant\ known\ as\ frequency\ sensitivity\ of\ the\ modulator\ (Hz/Volt)$ and m(t) is the message signal.

OR

$f_i = f_c + k_f m(t)$ .... (6) **THIS IS THE DEFINITION OF FM**

Equation **6** says that something must be added to fc to get fi. Because, as per definition of FM, frequency of carrier must change.

Substitute equation **6** into equation **4**, we get

$$\therefore\ d\theta_i = 2\pi f_i dt = 2\pi\{f_c + k_f\, m(t)\}\, dt$$
$$= 2\pi f_c\, dt + 2\pi\, k_f\, m(t) dt$$

$\therefore d\theta_i = 2\pi f_c\, dt + 2\pi\, k_f\, m(t) dt$ .... (7)

Taking integration on bothe sides of equation **7**,

$$\int d\theta_i = \int \{2\pi f_c\, dt + 2\pi\, k_f\, m(t) dt\}$$

$$\theta_i = \int 2\pi f_c\, dt + \int 2\pi\, k_f\, m(t) dt$$

$$= 2\pi f_c\, t + 2\pi k_f \int m(t)\; dt$$

$$\therefore\; \theta_i = \; 2\pi f_c\, t + 2\pi k_f \int m(t)\; dt \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (8)$$

Now sustituting equation **8** into equation **2**, we get

$$FM(t) = \; A_c \cos[w_c t + \; k_f \; \int m(t)\, dt \;\ldots\ldots\ldots\ldots\ldots\ldots \quad (9)$$

Now, if the phase angle of the carrier (un-modulated carrier) is taken at t = 0, then the limit of integration in equation 5 will be 0 to t.

∴ The general expression for an FM wave will be:

$$FM(t) = \; A_c \cos[w_c t + \; k_f \int_0^t m(t)\, dt$$

where

Modulating signal **m(t) = baseband signal**
= intelligence signal
= information bearing signal
= message signal
= input signal

Note that m(t) can take any shape. For mathematical analysis, we assume m(t) as sinusoidal (sine wave Or cosine wave)

Single-tone FM

Single-tone means that input signal consists of one frequency component. If the input signal contains more than one frequency, is known as multi-tone. Example of multi-tone signal is voice (speech) signal, which contain any frequency from 20 Hz to 20 KHz.

We know that general expression for FM wave is:

$$FM(t) = \; A_c \cos[w_c t + \; k_f \int_0^t m(t)\, dt$$

For single-tone input assume, $m(t) = \; A_m \cos w_m\, t$

$$\therefore\; FM(t) = \; A_c \cos[w_c t + \; k_f \int_0^t A_m \cos w_m\, dt$$

$$FM(t) = \; A_c \cos[w_c t + \frac{k_f A_m}{w_m} \sin w_m\, t]$$

$$k_f A_m = \; \Delta_f = Frequencu\ deviation$$

$$\&\ modulation\ index\ \; m_f = \frac{\Delta_f}{f_m} = \frac{\Delta_w}{w_m}$$

Therefore, the required expression for single-tone FM wave will be:

$$FM(t) = \; A_c \; \cos[w_c t + \; m_f \; \sin w_m\, t]$$

Some important terms related to FM are given below:

FREQUENCY DEVIATION (Δ_f)

The frequency deviation is useful in determining the FM signal bandwidth. Note that the deviation means change or variation. Δ_f is the maximum change in instantaneous frequency from fc (carrier) is called frequency deviation(Δ_f).

The amount of deviation i.e., variation depends upon the amplitude (loudness) of the modulating signal (i.e., audio signal). This means that louder the signal, greater the frequency deviation and vie-versa.

NOTE

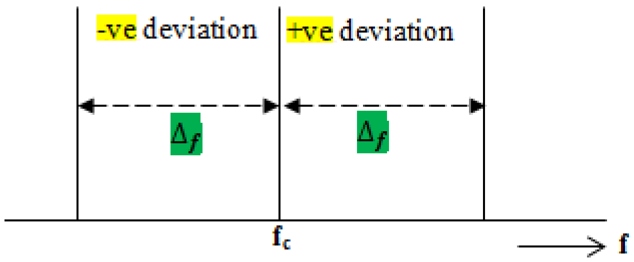
Change in instantaneous frequency w_i *from* f_c *depends upon the magnitude and sign of* $k_f\, m(t)$. Therefore frequency deviation $\Delta_f = |k_f m(t)|_{max} = k_f A_m$

The maximum change in instantaneous frequency from the carrier frequency (Δ_f) is called frequency deviation. Remember that a maximum frequency deviation of 75 KHz is allowed for commercial FM broadcasting (88 MHz – 108 MHz).

CARRIER SWING

The total variation in carrier frequency from the lowest to highest point is called carrier swing.

Carrier swing (in FM) = 2 x Δ_f



For commercial FM broadcast	→	$\Delta_f = 75\ kHz$
For TV broadcast	→	$\Delta_f = 25\ kHz$

- In FM broadcast, it has been internationally agreed to restrict maximum deviation to **75 kHz** on each side of carrier frequency (f_c).
- Frequency deviation (Δ_f) is related to LOUDENESS (= AMPLITUDE) of the modulating signal. Louder the sound, greater the (Δ_f) and vice-versa.

MODULATION INDEX (m_f)

For FM, the modulation index is defined as the ratio of frequency deviation (Δ_f) to the modulating frequency (f_m).

$$m_f = \frac{\Delta_f}{f_m} = \frac{Frequency\ deviation}{Modulating\ frequency}$$

NOTE: in FM broadcast, the highest frequency transmitted is 15 kHz.

$\therefore\ f_m$ = Maximum frequency of audio signal
= 15 kHz
= Maximum modulating frequency
= maximum input frequency

PERCENT MODULATION

Refers to the ratio of actual frequency deviation (Δ_f) to the maximum allowable frequency deviation $(\Delta_f)_{max}$

$$\therefore\ Percent\ Modulation = \%\ M = \frac{(\Delta_f)_{actual}}{(\Delta_f)_{Max}}\ x\ 100$$

When $(\Delta_f)_{actual} = (\Delta_f)_{Max}, then\ M = 100\%$

- FM use in TV transmission: note that TV signal is a composite signal i.e., video + audio. Video is amplitude modulated (AM) and audio (sound) is frequency modulated (FM).

- FM is more immune to noise & interference than AM. So, performance of FM is good in the presence of noise.

Q1. A single-tone FM is represented by the voltage equation: $v(t) = 12\ \cos(6\ x\ 10^8 t + 5\ \sin 1250\, t)$. Determine the following:

i. Carrier frequency
ii. Modulation index
iii. Modulating frequency
iv. Maximum deviation
v. What power this FM wave will dissipate in 10 Ohm resistor

Sol. General expression for single-tone FM wave is

$$FM(t) = A_c\ \cos[w_c t + m_f\ \sin w_m\, t]$$

Given equation for FM is: $v(t) = 12\ \cos(6\ x\ 10^8 t + 5\ \sin 1250\, t$

Comparing above 2 equations, we get

$$\boldsymbol{A_c} = 12 \quad w_c = 6\ x\ 10^8\ rad/sec, \quad \boldsymbol{w_m} = 1250\ rad/sec$$

(i). Carrier frequency (f_c): $w_c = 2\pi f_c = 6\ x\ 10^8\ rad/sec$

$$\therefore\ f_c = \frac{6\ x\ 10^8}{2\pi} = 99.5\ MHz$$

(ii). Modulating frequency (f_m):

$$w_m = 2\pi f_m = 1250\, rad/sec$$

$$\therefore\ f_m = \frac{1250}{2\pi} = 199\ Hz$$

(iii). Modulation index, $m_f = 5$

(iv). Maximum deviation = frequency deviation = Δ_f

We know that $m_f = \frac{\Delta_f}{f_m}$

$$5 = \frac{\Delta_f}{199} \quad \therefore \Delta_f = 5\ x\ 199 = 995\ Hz$$

(v) Power dissipation

P = $P = I_{rms}^2\ R = \frac{V_{rms}^2}{R}$

R = resistance = 10 Ohm

$V_{rms} = \frac{12}{\sqrt{2}}$ = 8.485 volts

$$P = \frac{V_{rms}^2}{R} = \frac{(8.485)^2}{10} = 7.2\ Watts$$

NOTE: Since the amplitude of the FM signal remains unchanged, the power of FM signal will be same as that of the un-modulated carrier.

Q2. Find the instantaneous frequency in Hz of each of the following signals:

i. $10\ \cos\left(200\pi t + \frac{\pi}{3}\right)$

ii. $10\ \cos(20\pi t + \pi t^2)$

iii. $\cos 200\pi t\ .\cos(5\sin 2\pi t) + \sin 200\pi t\ .\sin\ (5\sin 2\pi t)$

Sol.

$(i). 10\ \cos(\underbrace{200\pi t + \frac{\pi}{3}}_{\theta_i})$

$\theta_i \rightarrow$ phase, which is function of t

$$FM(t) = A_c\cos(w_i t) = A_c\cos 2\pi f_i t = A_c\cos\theta_i$$

$$\Rightarrow \theta_i = 2\pi f_i t$$

Differentiating both sides with respect to t, $\frac{d\theta_i}{dt} = 2\pi f_i$

$$\therefore 2\pi f_i = \frac{d\theta_i}{dt} = \frac{d}{dt}\left\{200\pi t + \frac{\pi}{3}\right\}$$

$$= \frac{d}{dt}\{200\pi t\} + \frac{d}{dt}\left(\frac{\pi}{3}\right) = 200\pi = 2\pi\ x\ 100 = 2\pi\ x\ f_i$$

$$\therefore\ f_i = instantaneous\ frequency = 100\ Hz$$

$(ii). 10\ \cos(\underbrace{20\pi t + \pi t^2}_{\theta_i})$

θ_i = phase angle of modulated carrier wave

$$\therefore\ \theta_i = 20\pi t + \pi t^2$$

$$2\pi f_i = \frac{d}{dt}\theta_i = \frac{d}{dt}\{20\pi t + \pi t^2\} = \frac{d}{dt}20\pi t + \frac{d}{dt}\pi t^2 = \mathbf{20\pi + \pi\ x\ 2t = 2\pi(10 + t)}$$

$$\therefore\ f_i = 10 + t$$

$at\ t = 0 \longrightarrow f_i = 10$

$at\ t = 1 \longrightarrow f_i = 11$

$at\ t = 2 \longrightarrow f_i = 12$ ……………. & increases linearly at a rate of 1 Hz/sec

(iii). $\cos 200\pi t\ .\cos(5\sin 2\pi t) + \sin 200\pi t\ .\sin\ (5\sin 2\pi t)$

= Cos A. Cos B + Sin A. Sin B

= cos (A-B)

$= \cos(\underbrace{200\pi t - 5\sin 2\pi t}_{\theta_i})$

θ_i = phase angle of modulated carrier wave

$$2\pi f_i = \frac{d}{dt}\theta_i = \frac{d}{dt}\{200\pi t - 5\ sin 2\pi t\} = \mathbf{200\pi - 5\cos 2\pi t\ x\ 2\pi = 2\pi\ (100 - 5\cos 2\pi t)}$$

$$\therefore\ f_i = 100 - 5\cos 2\pi t$$

$at\ t = 0 \longrightarrow f_i = 100 - 5 = 95\ Hz$

$at\ t = 90 \longrightarrow f_i = 100\ Hz$

Q3. A 107.6 MHz carrier signal is frequency modulated by a 7 kHz sine wave. The resultant FM signal has a frequency deviation of 50 kHz. Determine the following:

i. Carrier swing of FM signal
ii. Highest & lowest frequencies attained by the modulated signal
iii. The modulation index of FM wave

Sol.

$Given\ data\ f_c = 107.6\ MHz, f_m = 7\ kHz, \Delta_f = 50\ kHz$

$(i).\ carrier\ swing = 2\ x\ \Delta_f = 2\ x\ 50 = 100\ kHz$

$(ii).\ f_H = f_c + \Delta_f = highest\ frequency\ attained\ by\ modulated\ singal\ (FM\ signal)$

= 107.6 MHz + 50 kHz = 107.65 MHz

$f_L = f_c - \Delta_f = lowest\ frequency\ attained\ by\ modulated\ singal\ (FM\ signal)$

= 107.6 MHz – 50 kHz = 107.55 MHz

$$(iii).\ Modulation\ index, m_f = \frac{\Delta_f}{f_m} = \frac{50\ kHz}{7\ kHz} = 7.143$$

Q4. What is the modulation index of an FM signal having a carrier of 100 kHz when the modulating signal has a frequency of 8 kHz?

Sol.

$carrier\ swing = 2\ x\ \Delta_f = 1000\ kHz \longrightarrow \Delta_f = 50\ kHz$

Modulating frequency, $f_m = 8\ kHz$

$$modulation\ index\ m_f = \frac{\Delta_f}{f_m} = \frac{50\ kHz}{8\ kHz} = 6.25$$

Q5. An FM transmission has a frequency deviation of 20 kHz. Determine:

i. % modulation of this signal if it is broadcasted in the 88 – 108 MHz band.
ii. Calculate the % modulation if this signal is broadcasted as the audio portion of a TV broadcast

Sol.

$(i)\ For$ FM broadcast$(\Delta_f)_{max} = 75\ kHz$

$$\%\ M = \frac{(\Delta_f)_{actual}}{(\Delta_f)_{max}} = \frac{20\ kHz}{75\ kHz} = 26.67\ \%$$

$(ii)\ For\ \ TV\ broadcast\ (\Delta_f)_{max} = 25\ kHz$

$$\%\ M = \frac{(\Delta_f)_{actual}}{(\Delta_f)_{max}} = \frac{20\ kHz}{25\ kHz} = 80\ \%$$

Types of FM

NBFM

- Range of frequency operation is very narrow

WBFM

- Operates on large range of frequencies

Characteristic	NBFM	WBFM
1. Modulation index	**<1**	**>1**
2. Maximum deviation Δ_f	5 kHz	75 kHz
3. Maximum modulation index (m_f)	Slightly > 1	5 to 2500
4. Modulating frequency range	30 Hz – 3 kHz	30 Hz – 15 kHz
5. BW	Small	LARGE approximately 15 times of AM
6. Applications	• FM mobile communications like: police wireless, ambulance etc • used for speech transmission	• Entertainment broadcasting • can be used for high quality music transmission

NOTE:

- Many of the advantages obtained with WBFM such as – noise reduction are not available with NBFM.
- BW of NBFM is almost same as that of AM (amplitude modulation)
- BW of WBFM ≅ 15 times that of AM.

CARSON's RULE

Is used to calculate the bandwidth (BW) of a single-tone wideband FM. Note that this rule calculates ***approximate*** value of the FM signal BW.

$$FM\ (BW) = 2(\Delta_f + f_m)$$

$$\Delta_f = frequency\ deviation$$

$$f_m = modulating\ frequency$$

We know that modulation index, $m_f = \frac{\Delta_f}{f_m} \longrightarrow \Delta_f = m_f f_m$

$$\therefore BW = 2(m_f f_m + f_m)$$
$$= 2 f_m\,(1 + m_f)$$

Case 1: NBFM: in this case $\Delta_f \ll f_m$

$$so, \Delta_f\ can\ be\ neglected$$

$$\therefore BW = 2(\Delta_f + f_m) = 2(0 + f_m) = 2\, f_m$$

$$\therefore For\ NBFM, BW\ \approx 2f_m = BW\ of\ AM$$

Case 2: WBFM: in this case $\Delta_f \gg f_m$

$$so, f_m\ can\ be\ neglected$$

$$\therefore BW = 2(\Delta_f + f_m) = 2(\Delta_f + 0) = 2\,\Delta_f$$

$$\therefore For\ NBFM, BW\ \approx 2f_m = BW\ of\ AM$$

Q. Find the bandwidth of a commercial FM transmission if frequency deviation Δ_f = 75 kHz and modulating frequency f_m = 15 kHz.

Sol. According to Carson's rule, BW is given by:

$$BW = 2(\Delta_f + f_m) = 2(75K + 15\,K) = 180\,KHz$$

Q. Determine the bandwidth of a narrow band FM signal which is generated by a 4KHz audio signal modulating a 125 MHz carrier.

Sol. $BW\ (NBFM) \approx 2f_m = 2x\ 4\ KHz = 8\ KHz$

Q. In a FM system, the modulating frequency, f_m = 1 KHz, the modulating voltage A_m = 2 volt and the deviation is 6 KHz. If the modulating voltage is raised to 4 volt, then what will be the new deviation? If the modulating voltage is further raised to 8 volt and the modulating frequency is reduced to 500 Hz, what will be new deviation?

Sol. Given data: f_m = 1 KHz, A_m = 2volt, Δ_f = 6 KHz

We know that $\Delta_f = k_f\, A_m$ where k_f = frequency sensitivity of modulator

$$\therefore\ k_f = \frac{\Delta_f}{f_m} = \frac{6KHz}{2\,V} = 3\ KHz/volt$$

Case 1: modulating voltage raised to 4 V

$A_{m1} = 4\,V$

New deviation $\Delta_f = k_f\, A_{m1} = 3\ x\ 4 = 12\ KHz$

Case 2: $A_{m2} = 4\,V\ and\ f_{m2} = 500\ Hz$

New Deviation $\Delta_f = k_f\, A_m = 3\ x\ 8 = 24\ KHz$

NOTE: change in modulating frequency has no effect on the deviation, because $\Delta_f = k_f\, A_m$

Q. For the same data of above question, calculate the modulation index in each case. Comment on the result.

Sol. Given data: f_m = 1 KHz, A_m = 2volt, Δ_f = 6 KHz, k_f = 3 KHz/volt

$$m_f = Modulation\ Index = \frac{\Delta_f}{f_m} = \frac{6\ KHz}{1\ KHz} = 6$$

Case 1: modulating voltage raised to 4V

$$\therefore\ \Delta_f = k_f\, A_m = 3\ x\ 4 = 12\ KHz$$

$$m_f = Modulation\ Index = \frac{\Delta_f}{f_m} = \frac{12\ KHz}{1\ KHz} = 12$$

Case 2: Δ_f = 24 KHz, f_m = 500 Hz

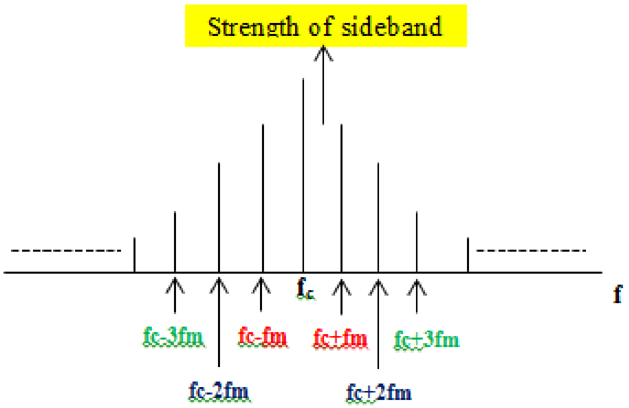
$$m_f = Modulation\ Index = \frac{\Delta_f}{f_m} = \frac{24\ KHz}{500\ Hz} = 48$$

Comment: modulation index depends on Δ_f as well as f_m.

FM SIDEBANDS

In FM, when a carrier is modulated, a number of sidebands are produced. Theoretically the number of sidebands is infinite, but their strength becomes negligible after few sidebands.

NOTE: for a single-tone AM, only 2 sidebands are produced. Sidebands of FM will lie on both sides of carrier frequency (f_c) spaced f_m apart as shown below.



If f_c is the carrier frequency, then f_m wave contains the following frequencies:

(i) fc
(ii) $f_c \pm f_m$
(iii) $f_c \pm 2f_m$ and so on.

So the actual bandwidth occupied by FM = 2nfm, where n = highest order of significant sideband

NOTE: Another approximate expression for spectrum bandwidth of FM is:

$BW = 2(\Delta_f + f_m)$ ⟸ Carson's rule.

Performance comparison of AM & FM systems

S.No.	FM	AM
1	Amplitude of FM wave is constant	Amplitude of AM will change with the modulating voltage
2	Transmitted power is independent of modulation index (mf)	Transmitted power depends on modulation index (ma)
3	All the transmitted power is useful	Carrier power and 1 SB power get wasted
4	FM receivers are immune to noise	AM receivers are not immune to noise
5	It is possible to decrease noise further by increasing Delta f	This feature is absent in AM
6	BW = $2(\Delta_f + f_m)$. Here BW depends on modulation index	BW = $2f_m$. It is not dependent on ma
7	BW required is large	BW is much less than FM
8	FM transmission & reception equipment are more complex	AM equipment are less complex
9	Number of sidebands having significant amplitudes depends on modulation index	Number of sidebands in AM is constant and equal to 2
10	The information is contained in the frequency variation of the carrier	The information is contained in the amplitude variation of the carrier

HOMEWORK PROBLEMS

Q1 An FM wave is given by $s(t) = 20\ \sin(6\ x\ 10^8 t + 7\ \sin 1250t\ )$. Determine

i. The carrier and modulating frequencies, the modulation index and the maximum deviation.
ii. Power dissipated by this FM wave in a 100 Ω resistor.

Q2 In an FM system, a 7 kHz baseband signal modulates 107.6 MHz carrier wave so that the frequency deviation is 50 kHz. Find

i. Carrier swing in the FM signal & modulation index
ii. The highest and lowest frequencies allowed by FM wave

Q3 A carrier is frequency modulated (FM) by a sinusoidal modulating signal x(t) of frequency 2 kHz and results in a frequency deviationΔ_f of 5 kHz. Find the bandwidth occupied by the FM waveform. The amplitude of the modulating sinusoid is increased by a factor 3 and its frequency lowered by 1 kHz. Find the new bandwidth.

> NOTE: when the amplitude of the modulating signal is tripled, then the frequency deviation (Δ_f) increases by 3 times.
>
> $$(\Delta_f)_{new} = 3 \text{ x } \Delta_f$$

Q4. A 5 kHz audio tune is used to modulate a 50 MHz carrier causing a frequency deviation of 20 kHz. Determine modulation index.

Q5. A FM wave is represented by the following equation: $v = 10\ \sin[5\ x\ 10^8\ t + 4\ \sin 1250\ t]$. Determine:

i. Carrier and modulating frequencies
ii. Modulation index and max deviation
iii. The power dissipated in this FM wave in a 5 Ω resistor