AM (Amplitude Modulation) Math Derivation

Modulation definition:

Modulation is the process by which some characteristic (**amplitude, frequency, or phase**) of the carrier is changed according to amplitude of the input (baseband signal). In case of voice signal, the value of amplitude depends on the LOUDENESS. The more loudly we speak, more the amplitude value.

- In **Amplitude Modulation (AM)**, amplitude of the carrier is changed in accordance with amplitude of modulating signal.
- In **Frequency Modulation (FM)**, frequency of the carrier is changed in accordance with amplitude of modulating signal.
- In **Phase Modulation (PM)**, phase of the carrier is changed in accordance with amplitude of modulating signal.

- Transmitter modifies the message signal in order to transport information easily from one place to other. This modification is called modulation.
- During this process, Low Frequency (LF) signal changes the High Frequency (HF) signal.
- By modulation, baseband signal is translated from Low Frequency (LF) to High Frequency (HF).
- Modulation is the process of adding carrier to modulating signal. Demodulation is the process of removing carrier from modulating signal.

Mathematical Analysis of single –tone AM

- Real-life examples of ***baseband signal***: **voice, audio, music, video, computer data**

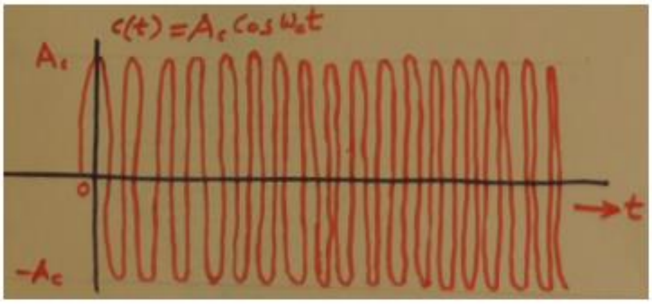
$$Let\ c(t) = A_c cos\omega_c t$$

c(t) = carrier signal

A_c = peak amplitude of carrier signal

ω_c = angular frequency of carrier (rad/sec)

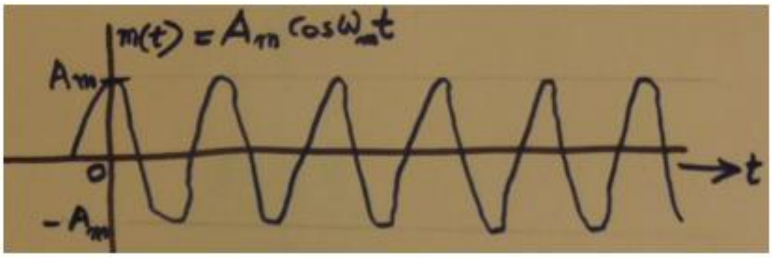
f_c= carrier frequency



AM (Amplitude Modulation)

$$Let\ m(t) = A_m cos\omega_m t$$

m(t) = Message signal
= Input signal
= Baseband signal
= Modulating signal
= Intelligent signal
= Information bearing signal



A_m = peak amplitude of baseband signal

ω_m = angular frequency of modulating signal

f_m = modulating signal frequency

According to definition of AM, we change the amplitude of the carrier

$$AM(t) = [A_c + m(t)]cos\omega_c t$$

$$= [A_c + A_m cos\omega_m t]\, cos\omega_c t$$

$$= A_c cos\omega_c t + A_m cos\omega_m t\, cos\omega_c t$$

By rearranging

$$AM(t) = A_c cos\omega_c t + \frac{1}{2}.2\, A_m cos\omega_m t\, cos\omega_c t$$

$$= A_c cos\omega_c t + \frac{A_m}{2}\, 2\, cos\omega_m t\, cos\omega_c t$$

$2\cos A\cos B = \cos(A+B) + \cos(A-B)$

$$= A_c cos\omega_c t + \frac{A_m}{2}[\cos(\omega_c+\omega_m)t + \cos(\omega_c-\omega_m)t]$$

$$= A_c cos\omega_c t + \frac{A_m}{2}\frac{A_c}{A_c}[\cos(\omega_c+\omega_m)t + \cos(\omega_c-\omega_m)t]$$

$$= A_c cos\omega_c t + \frac{A_c}{2}\frac{A_m}{A_c}[\cos(\omega_c+\omega_m)t + \cos(\omega_c-\omega_m)t]$$

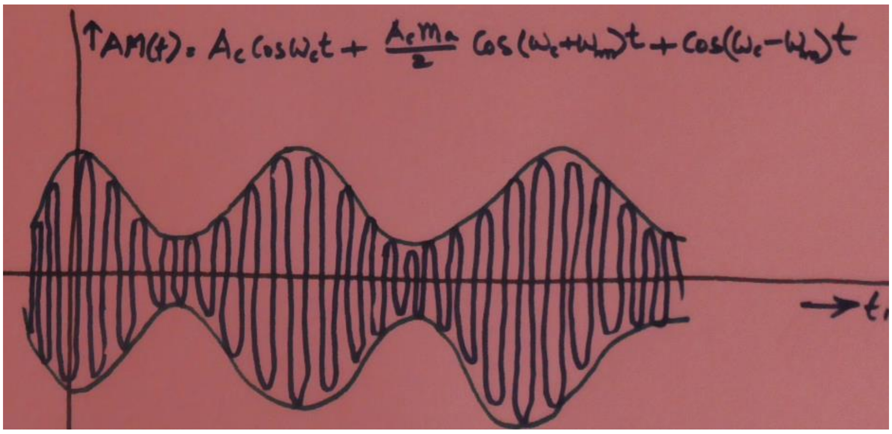
$$= A_c cos\omega_c t + \frac{A_c m_a}{2}\cos(\omega_c+\omega_m)t + \frac{A_c m_a}{2}\cos(\omega_c-\omega_m)t$$

$$= A_c cos\omega_c t + \frac{A_c m_a}{2}\cos 2\pi(f_c+f_m)t + \frac{A_c m_a}{2}\cos\, 2\pi(f_c-f_m)t$$

carrier USB LSB

where m_a = modulation index $= \frac{A_m}{A_c}$

AM Waveform



Observe that modulated waveform contains 3 frequency components.

1. Carrier
2. USB (Upper Side Band)
3. LSB (Lower Side Band)

$$A_c cos\omega_c t + \frac{A_c m_a}{2}\cos 2\pi(f_c+f_m)t + \frac{A_c m_a}{2}\cos\, 2\pi(f_c-f_m)t$$

carrier USB LSB

AM Derivation Practice

Q1. $Assume\ m(t) = A_m sin\omega_m t\ and\ c(t) = A_c sin\omega_c t$

Derive expression for amplitude modulated (AM) wave AM(t).

Answer: $A_c sin\ \omega_c t + \frac{A_c m_a}{2} \cos 2\pi(f_c - f_m)t - \frac{A_c m_a}{2} \cos\ 2\pi(f_c + f_m)t$

Q2. $Assume\ m(t) = A_m sin\omega_m t\ and\ c(t) = A_c cos\omega_c t$

Derive expression for amplitude modulated (AM) wave AM(t).

Answer: $A_c cos\ \omega_c t + \frac{A_c m_a}{2} \sin 2\pi(f_c - f_m)t - \frac{A_c m_a}{2} \sin\ 2\pi(f_c + f_m)t$

Q3. $Assume\ m(t) = A_m cos\omega_m t\ and\ c(t) = A_c sin\omega_c t$

Derive expression for amplitude modulated (AM) wave AM(t).

Answer: $A_c sin\ \omega_c t + \frac{A_c m_a}{2} \sin 2\pi(f_c + f_m)t + \frac{A_c m_a}{2} \sin\ 2\pi(f_c - f_m)t$